

Roll No. \_\_\_\_\_

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4E1301

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**B.Tech. IV Sem. (Main/Back) Examination, July - 2023**  
**Artificial Intelligence and Data Science**  
**4AID2-01 Discrete Mathematics Structure**  
**CS, IT, AID, CAI**

Time : 3 Hours

Maximum Marks : 70

**Instructions to Candidates:**

*Attempt all ten questions from Part A, Attempt any five questions out of Seven questions from Part B and three questions out of Five questions from Part C.*

*Schematic diagrams must be shown wherever necessary. Any data missing may suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.*

*Use of following supporting material is permitted during examination. (As Mentioned in form No. 205).*

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**PART - A**

(Word limit 25)

**All questions are compulsory.****(10×2=20)**

1. Prove that  $A - B = B' \cap A'$ . (2)
2. Define partial ordering relation with an example. (2)
3. Draw the truth table for  $p \wedge q \rightarrow p \vee q$ . (2)
4. Explain conjunctive normal form. (2)
5. Draw the Hasse diagram of the poset  $(A, \leq)$  where  $A = \{1, 2, 3, 4, 12\}$  and the partial order of divisibility on A is  $a \leq b$  (i.e. if a divides b). (2)
6. 8 boys and 5 girls constitute a group. In how many ways seven of them can be selected if the selections always have atleast 3 boys and 2 girls. (2)
7. If a, b are any elements of a group G, then prove that  $(ab)^{-1} = b^{-1}a^{-1}$ . (2)
8. If a is a generator of a cyclic group, then prove that  $a^{-1}$  is also its generator. (2)
9. Draw graph which is
  - a. Eulerian but not Hamiltonian.
  - b. Hamiltonian but not Eulerian. (2)
10. Define chromatic number. (2)

**PART - B**  
(Word limit 100)

Attempt any Five questions.

(5×4=20)

1. In a class of 60 students, 25 study Hindi, 26 study English and 26 study Sanskrit. Also 9 study both Hindi and Sanskrit, 11 study hindi and english and 8 study Englishh and Sanskrit. If 8 study none of the three subjects, Find number of students who study exactly one subject. (4)
2. Using principle of mathematical induction, prove that  $1 + 2 + 2^2 + \dots + 2^{n-1} = 2^n - 1$ . (4)
3. Draw the transition diagram of finite state machine represented by following state table. Also find output word corresponding to input word  $w = 11011011$ . (4)

State	Transition	function	Output	function
	0	1	0	1
$s_0$	$s_3$	$s_1$	0	1
$s_1$	$s_0$	$s_1$	0	1
$s_2$	$s_3$	$s_1$	0	1
$s_3$	$s_1$	$s_3$	0	0

4. Solve the recurrence relation

$$a_r - 4a_{r-1} + 4a_{r-2} = 0 ; a_0 = 1, a_1 = 3.$$

(4)

5. Define isomorphism of groups. Prove that every subgroup of an abelian group is normal. (4)
6. Prove that the sum of degrees of all the vertices in a graph is equal to twice the number of edges in the graph. (4)
7. Prove that set of real numbers of the form  $a + b\sqrt{2}$  (where a and b are integers) with ordinary addition and multiplication forms a ring. (4)

**PART - C**

Attempt any Three questions.

(3×10=30)

1. In the set Z of integers, a relation R is defined by  $aRb \Leftrightarrow a \equiv b \pmod{4}$ . Show that R is an equivalence relation. (10)
2. Define universal and existential quantifiers. prove that  $p \rightarrow (q \wedge r) \equiv (p \rightarrow q) \wedge (p \rightarrow r)$ . (10)
3. Prove that the dual of a lattice is also a lattice. (10)

4. Prove that the necessary and sufficient conditions for a non - void subset  $H$  of a group  $G$  to be a subgroup is that  $a \in H, b \in H \Rightarrow ab^{-1} \in H$ . (10)
5. Find the shortest path between the vertices  $v_0$  and  $v_3$  in the following weighed graph. (10)

